ABSTRACTS

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(1) T. Gateva–Ivanova, *Set-theoretic solutions of the Yang-Baxter equation, Braces, and Symmetric groups*, arXiv preprint, arXiv:1507.02602, 9 Jul 2015, 1–47

Abstract. Set-theoretic solutions of the Yang–Baxter equation form a meeting-ground of mathematical physics, algebra and combinatorics. Such a solution (X, r) consists of a set X and a bijective map $r : X \times X \to X \times X$ which satisfies the braid relations. In this work we involve simultaneously the matched pairs of groups theory and the theory of braces to study set-theoretic solutions of YBE. We show the intimate relation between the notions of a symmetric group (a braided involutive group) and a left brace, and find new results on symmetric groups of finite multipermutation level and the corresponding braces. We prove that a square-free solution (X, r) has finite multipermutation level $\operatorname{mpl} X = m$ *iff* the associated symmetric group $G(X, r)$ does so. We find new criteria sufficient to claim that a symmetric set (X, r) is a multipermutation solution. In particular, we show that if (X, r) is a finite solution with permutation group $\mathcal{G} = \mathcal{G}(X, r)$ then each of the conditions (i) $\mathcal{G}$ is a two-sided brace; (ii) (X, r) is square-free and $\mathcal{G}$ has mutually inverse left and right actions upon itself (condition **lri**) is sufficient to claim that (X, r) is a multipermutation solution. Moreover, if (X, r) is a finite square-free solution, and $\mathcal{G}$ is a two-sided brace, then $\operatorname{mpl} X$ equals the nilpotency degree of the radical ring associated with $\mathcal{G}$. We prove that each solutions (X, r) whose associated brace $(G, +, .)$ is two-sided must be a trivial solution.

(2) T. Gateva–Ivanova, G. Floystad, *Monomial algebras defined by Lyndon words*, J. Algebra **403**(2014)470-496 ISSN: 0021-8693 **Impact factor** 0.599; 5 years Impact factor 0.698

Abstract. Assume that $X = \{x_1, \cdots, x_g\}$ is a finite alphabet and K is a field. We study monomial algebras $A = K\langle X\rangle/(W)$, where W is an antichain of Lyndon words in X of arbitrary cardinality. We find a Poincaré-Birkhoff-Witt type basis of A in terms of its *Lyndon atoms* N, but, in general, N may be infinite. We prove that if A has polynomial growth of degree d then A has global dimension d and is standard finitely presented, with $d - 1 \leq |W| \leq d(d - 1)/2$. Furthermore, A has polynomial growth *iff* the set of Lyndon atoms N is finite. In this case A has a K-basis $\mathfrak{N} = \{l_1^{\alpha_1} l_2^{\alpha_2} \cdots l_d^{\alpha_d} \mid \alpha_i \geq 0, 1 \leq i \leq d\}$, where $N = \{l_1, \cdots, l_d\}$. We give an extremal class of monomial algebras, the Fibonacci-Lyndon algebras, F_n, with global dimension n and polynomial growth, and show that the algebra F_6 of global dimension 6 cannot be deformed, keeping the multigrading, to an Artin-Schelter regular algebra.

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(3) T. Gateva-Ivanova, *Quadratic algebras, Yang–Baxter equation, and Artin–Schelter regularity*, Adv. in Math. **230** (2012), 2152-2175. ISSN: 0001-8708 **Impact factor** 1.373; **5 years Impact factor** 1.510

Abstract. We study quadratic algebras over a field $\mathbf{k}$. We show that an n-generated PBW algebra A has finite global dimension and polynomial growth *iff* its Hilbert series is $H_A(z) = 1/(1-z)^n$. Surprising amount can be said when the algebra A has *quantum binomial relations*, that is the defining relations are nondegenerate square-free binomials $xy - c_{xy}zt$ with non-zero coefficients $c_{xy} \in \mathbf{k}$. In this case various good algebraic and homological properties are closely related. The main result shows that for an n-generated quantum binomial algebra A the following conditions are equivalent: (i) A is a PBW algebra with finite global dimension; (ii) A is PBW and has polynomial growth; (iii) A is an Artin-Schelter regular PBW algebra; (iv) A is a Yang-Baxter algebra; (v) $H_A(z) = 1/(1-z)^n$; (vi) The dual $A^!$ is a quantum Grassman algebra; (vii) A is a binomial skew polynomial ring. So for quantum binomial algebras the problem of classification of Artin-Schelter regular PBW algebras of global dimension n is equivalent to the classification of square-free set-theoretic solutions of the Yang-Baxter equation (X, r), on sets X of order n.

(4) T. Gateva–Ivanova, Peter Cameron, *Multipermutation solutions of the Yang–Baxter equation* , Comm. Math. Phys, **309** (2012), 583-621. ISSN: 0010-3616 (Print) 1432-0916 (Online) **Impact factor** 1.971; **5 years Impact factor** 2.012

Abstract. Set-theoretic solutions of the Yang–Baxter equation form a meeting-ground of mathematical physics, algebra and combinatorics. Such a solution consists of a set X and a function $r : X \times X \to X \times X$ which satisfies the braid relation. We examine solutions here mainly from the point of view of permutation groups: a solution gives rise to a map from X to the symmetric group $\mathrm{Sym}(X)$ on X satisfying certain conditions, whose image we call a Yang–Baxter permutation group. Our results include new constructions based on strong twisted unions, with an investigation of retracts and the multipermutation level and the solvable length of the groups defined by the solutions; new results about decompositions of solutions of arbitrary cardinality into invariant subsets and decompositions and factorisations of the associated Yang–Baxter group as a product of groups of the solutions defined by these invariant subsets. In particular, we obtain strong decomposition results if the Yang–Baxter permutation group is abelian or the solution is of finite multipermutation level.

(5) T. Gateva–Ivanova, *Garside structures on monoids with quadratic square-free relations* , Algebr. Represent. Theor. **14** (2011) 779-802, Springer, ISSN 1572-9079 **Impact factor** 0.595; **5 years Impact factor** 0.622

Abstract. We show the intimate connection between various mathematical notions that are currently under active investigation: a class of *Garside monoids*, with a nice Garside element, certain monoids S with quadratic relations, whose monoidal algebra $\mathcal{A} = kS$ has *a Frobenius Koszul dual* $\mathcal{A}'$ with *regular socle*, *the monoids of skew-polynomial type* (or equivalently, *binomial skew-polynomial rings*) which were introduced and studied by the author and in 1995 provided a new class of Noetherian *Artin-Schelter regular domains*, and *the square-free set-theoretic solutions of the Yang- Baxter equation.* There is a beautiful symmetry in these objects due to their nice combinatorial and algebraic properties.

(6) T. Gateva-Ivanova, S. Majid, *Quantum spaces associated to multipermutation solutions of level two*, Algebr. Represent. Theor., **14** (2011) 341-376, Springer, ISSN 1572-9079 **Impact factor** 0.595; **5 years Impact factor** 0.522

Abstract. We study finite set-theoretic solutions (X, r) of the Yang-Baxter equation of square-free multipermutation type. We show that each such solution over $\mathbb{C}$ with multipermutation level two can be put in diagonal form with the associated Yang- Baxter algebra $\mathcal{A}(\mathbb{C}, X, r)$ having a q-commutation form of relations determined by complex phase factors. These complex factors are roots of unity and all roots of a prescribed form appear as determined by the representation theory of the finite abelian group $\mathcal{G}$ of left actions on X. We study the structure of $\mathcal{A}(\mathbb{C}, X, r)$ and show that they have a $\bullet$-product form quantizing the commutative algebra of polynomials in $|X|$ variables. We obtain the $\bullet$-product both as a Drinfeld cotwist for a certain canonical 2-cocycle and as a braided-opposite product for a certain crossed $\mathcal{G}$-module (over any field k). We provide first steps in the noncommutative differential geometry of $\mathcal{A}(\mathbb{C}, X, r)$ arising from these results. As a byproduct of our work we find that every such level 2 solution (X, r) factorises as $r = f \circ \tau \circ f^{-1}$ where τ is the flip map and (X, f) is another solution coming from X as a crossed $\mathcal{G}$-set.

(7) Gateva–Ivanova, T. and Cameron, P.J., *Multipermutation solutions of the Yang–Baxter equation*, Arxiv arXiv:0907.4276v1 [math.QA] 24 Jul 2009, 46–60, Sections 8,9, 10.

Abstract. We define the notion of *wreath product of square-free solutions*, by analogy with the wreath products of permutation groups. We prove that the wreath product $(Z, r) = (X, r_X) \operatorname{wr} (Y, r_Y)$ of two multipermutation square-free solutions is also a multipermutation solution with $\operatorname{mpl} Z = \operatorname{mpl} X + \operatorname{mpl} Y - 1$. We construct an interesting sequence of explicitly defined square-free solutions $(X_m, r_m), m = 0, 1, 2, ...,$ such that $\operatorname{mpl}(X_m) = m$, each retract $\operatorname{Ret}(X_{m+1}, r_{m+1})$ is isomorphic to the solution (X_m, r_m), and $m = G(X_m, r_m) = \mathcal{G}(X_m, r_m) + 1$.

These results are not included in the paper with the same title in Comm. Math. Phys. 2012. and will be published in a separate paper.

(8) T. Gateva-Ivanova, S. Majid, *Matched pairs approach to set theoretic solutions of the Yang–Baxter equation*, J. Algebra, **319** (2008) 1079–1112. ISSN: 0021-8693 **Impact factor** 0.630; **5 years Impact factor** 0.668

ABSTRACT. We study set-theoretic solutions (X, r) of the Yang-Baxter equations on a set X in terms of the induced left and right actions of X on itself. We give a characterization of involutive square-free solutions in terms of cyclicity conditions. We characterise general solutions in terms of abstract matched pair properties of the associated monoid $S(X, r)$ and we show that r extends as a solution r_S on $S(X, r)$ as a set. Finally, we study extensions of solutions both directly and in terms of matched pairs of their associated monoids. We also prove several general results about matched pairs of monoids S of the required type, including iterated products $S \bowtie S \bowtie S$ equivalent to r_S a solution, and extensions $(S \bowtie T, r_{S \bowtie T})$. Examples include a general 'double' construction $(S \bowtie S, r_{S \bowtie S})$ and some concrete extensions, their actions and graphs based on small sets.

(9) T. GATEVA–IVANOVA, S. MAJID, *Set theoretic solutions of the Yang–Baxter equation, graphs and computations*, J. SYMBOLIC COMPUTATION, **42** (**2007**), 1079–1112. ISSN: 0747-7171 **Impact factor** 0.658; **5 years Impact factor** 0.739

ABSTRACT. We extend our recent work on set-theoretic solutions of the Yang–Baxter or braid relations with new results about their automorphism groups, strong twisted unions of solutions and multipermutation solutions. We introduce and study graphs of solutions and use our graphical methods for the computation of solutions of finite order and their automorphisms. Results include a detailed study of solutions of multipermutation level 2.

(10) T. GATEVA-IVANOVA, *A combinatorial approach to the set-theoretic solutions of the Yang–Baxter equation*, J. MATH. PHYSICS, **45**, (2004), 3828 –3858. ISSN: 0022-2488 E-ISSN: 1089-7658 **Impact factor** 1.430

ABSTRACT. A bijective map $r : X^2 \longrightarrow X^2$, where $X = \{x_1, x_2, \cdots, x_n\}$ is a finite set, is called a settheoretic solution of the YangBaxter equation (YBE) if the braid relation $r^{12}r^{23}r^{12} = r^{23}r^{12}r^{23}$ holds in X^3. A nondegenerate involutive solution (X, r) satisfying $r(xx) = xx$, for all $x \in X$, is called a square-free solution. There exist close relations between the square-free set-theoretic solutions of YBE, the semigroups of I-type, the semigroups of skew polynomial type, and the Bieberbach groups, as it was first shown in a joint paper with Michel Van den Bergh. In this paper we continue the study of square-free solutions (X, r) and the associated YangBaxter algebraic structuresthe semigroup $S(X, r)$, the group $G(X, r)$ and the $\mathbf{k}$-algebra $\mathcal{A}(\mathbf{k}, X, r)$ over a field $\mathbf{k}$, generated by X and with quadratic defining relations naturally arising and uniquely determined by r. We study the properties of the associated Yang- Baxter structures, and prove a conjecture of the present author that the three notions: a square-free solution of (set-theoretic) YBE, a semigroup of I type, and a semigroup of skew-polynomial-type, are equivalent. This implies that the Yang Baxter algebra $\mathcal{A}(\mathbf{k}, X, r)$ is a PoincaréBirkhoffWitt-type algebra, with respect to some appropriate ordering of X. We conjecture that every square-free solution of YBE is retractable, in the sense of Etingof-Schedler-Solovyev.

(11) T. GATEVA-IVANOVA, *Binomial skew-polynomial rings, Artin–Schelter regular rings, and binomial solutions of the Yang–Baxter equation*, SERDICA MATH. J. **30** (2004), 431-470. ISSN 1310-6600

Abstract. Let $\mathbf{k}$ be a field and X be a set of n elements. We introduce and study a class of quadratic $\mathbf{k}$-algebras called quantum binomial algebras. Our main result shows that such an algebra $\mathcal{A}$ defines a solution of the classical Yang–Baxter equation (YBE), if and only if its Koszul dual $\mathcal{A}^!$ is Frobenius of dimension n, with a regular socle and for each $x, y \in X$ an equality of the type $xyy = \alpha zzt$, where $\alpha \in \mathbf{k}^\times$, and $z, t \in X$, is satisfied in $\mathcal{A}$. We prove the equivalence of the notions 'a binomial skew polynomial ring' and 'a binomial solution of YBE'. This implies that the Yang–Baxter algebra of such a solution is of Poincaré-Birkhoff-Witt type, and possesses a number of other nice properties such as being Koszul, Noetherian, and an Artin–Schelter regular domain.

(12) T. Gateva-Ivanova, E. Jespers, and J. Okniński, *Quadratic algebras of skew type and the underlying semigroups*, J. Algebra, **270** (2003), 635–659, Elsevier. ISSN: 0021-8693

Abstract. We consider algebras over a field K defined by a presentation $K\langle x_1, ..., x_n | R\rangle$, where R consists of $\binom{n}{2}$ square-free relations of the form $x_i x_j = x_k x_l$ with every monomial $x_i x_j, i \neq j$, appearing in one of the relations. Certain sufficient conditions for the algebra to be noetherian and PI are determined. For this, we prove more generally that right Noetherian algebras of finite Gelfand Kirillov dimension defined by homogeneous semigroup relations satisfy a polynomial identity. The structure of the underlying monoid, defined by the same presentation, is described. This is used to derive information on the prime radical and minimal prime ideals. Some examples are described in details. Earlier, Gateva-Ivanova and van den Bergh, and Jespers and Okniński considered special classes of such algebras in the contexts of noetherian algebras, Gröbner bases, finitely generated solvable groups, semigroup algebras, and set theoretic solutions of the YangBaxter equation.

(13) T. Gateva-Ivanova, *Set theoretic solutions of the Yang–Baxter equation*, Mathematics and education in Mathematics, Proc. of the Twenty Ninth Spring Conference of the Union of Bulgarian Mathematicians, Lovetch (2000), 107–117.

Abstract. The paper considers some most recent results on a class of solutions of the now famous Yang–Baxter equation, the so-called set-theoretic solutions. Our approach is algebraic. We discuss also our conjecture on the close relation between the nondegenerate involutive solutions of the set-theoretic Yang–Baxter equation and a class of standard finitely presented semigroups called binomial skew-polynomial semigroups

(14) T. Gateva-Ivanova, M. Van den Bergh, *Semigroups of I-type*, J. Algebra, **206** (1998), 97–112, Elsevier. ISSN: 0021-8693 **Impact factor** 0.422;

ABSTRACT. Assume that S is a semigroup generated by $\{x_1, \dots, x_n\}$, and let $\mathcal{U}$ be the multiplicative free commutative semigroup generated by $\{u_1, \dots, u_n\}$. We say that S is of *I-type* if there is a bijection $v : \mathcal{U} \longrightarrow S$ such that for all $a \in \mathcal{U}$, $\{v(u_1a), \dots v(u_na)\} = \{x_1v(a), \dots, x_nv(a)\}$. This condition appeared naturally in the work on Sklyanin algebras by John Tate and the second author. In this paper we show that the condition for a semigroup to be of I-type is related to various other mathematical notions found in the literature. In particular we show that semigroups of I-type appear in the study of the settheoretic solutions of the Yang-Baxter equation, in the theory of Bieberbach groups and in the study of certain skew binomial polynomial rings which were introduced by the first author.

(15) T. GATEVA-IVANOVA, *Skew polynomial rings with binomial relations*, J. ALGEBRA, **185** (1996), 710-753, ELSEVIER. ISSN: 0021-8693 **Impact factor** 0.473

ABSTRACT. In this paper we continue the study of a class of standard finitely presented quadratic algebras A over a fixed field K, called binomial skew polynomial rings. We consider some combinatorial properties of the set of defining relations F and their implications for the algebraic properties of A. We impose a condition, called $(*)$, on F and prove that in this case A is a free module of finite rank over a strictly ordered Noetherian domain. We show that an analogue of the Diamond Lemma is true for one-sided ideals of a skew polynomial ring A with condition $(*)$. We prove, also, that if the set of defining relations F is square free, then condition $(*)$ is necessary and sufficient for the existence of a finite Groebner basis of every one-sided ideal in A, and for left and right Noetherianness of A. As a corollary we find a class of finitely generated non-commutative semigroups which are left and right Noetherian.

(16) T. GATEVA-IVANOVA, *Noetherian properties of skew polynomial rings with binomial relations*, TRANS. AMER. MATH. SOC. **343** (1994), 203–219. ISSN 1088-6850(ONLINE) ISSN 0002-9947(PRINT) **Impact factor** 0.468;

ABSTRACT. In this work we study standard finitely presented associative algebras over a fixed field K. A restricted class of skew polynomial rings with quadratic relations considered in an earlier work of M. Artin and W. Schelter will be studied. We call them binomial skew polynomial algebras. We establish necessary and sufficient conditions for such an algebra to be a Noetherian domain

(17) GATEVA-IVANOVA, TATIANA *Noetherian properties and growth of some associative algebras.* EFFECTIVE METHODS IN ALGEBRAIC GEOMETRY (CASTIGLIONCELLO, 1990), 143-158, PROGR. MATH., **94**, BIRKHUSER BOSTON, BOSTON, MA, 1991. ISSN: 0079-6433

ABSTRACT. We consider finitely generated associative algebras over a fixed field K of arbitrary characteristic. For such an algebra A we impose some structural restrictions (we call A *strictly ordered*). We are interested in the implication of strict order on A for its Noetherian properties and type of growth. In particular, we prove that if A is a graded standard finitely presented strictly ordered algebra, then A is right (left) Noetherian if and only if A has polynomial growth. In this case A is almost commutative. It follows from this that the conjecture we made in [9] is true.

(18) T. GATEVA-IVANOVA, *On the Noetherianity of some associative finitely presented algebras*, J. ALGEBRA, **138** (1991), 13–35. ISSN: 0021-8693 **Impact factor** 0.402;

ABSTRACT. We consider finitely generated associative algebras over a fixed field K of arbitrary characteristic. For such an algebra A we impose some structural restrictions (We call A strictly ordered). We are interested in the implication of strict order on A for its Noetherian properties. In particular, we prove that if A is a graded standard finitely presented strictly ordered algebra, then A is left Noetherian if and only if it is almost commutative. In this case A has polynomial growth.

(19) T. GATEVA-IVANOVA, *Global dimension of associative algebras*, LECT. NOTES COMP. SCI., **357** (**1989**), SPRINGER BERLIN HEIDELBERG, 1989 213–229. ISSN: 0302-9743

ABSTRACT. Let G be a connected graded s.f.p. (standard. finitely presented) associative algebra over a field $\mathbf{k}$. We show that the g1oba1 dimension of G is effectively computable in the following cases: 1) G is a finitely presented monomial algebra; 2) G is a connected graded standard finitely presented algebra and the associated monomial algebra $A(G)$ has finite global dimension. The situation is considerably simpler when G has *polynomial growth of degree d* and $gl.\dim A(G) < \infty$. We show that in this case $gl.\dim G = gl.\dim A(G) = d$.

(20) T. GATEVA-IVANOVA, V. LATYSHEV, *On recognizable properties of associative algebras*, J.SYMB.COMP., **6** (1989) 371–398, ELSEVIER ISSN: 0747-7171 **Impact factor 0.904**

IBID. T. GATEVA-IVANOVA, V. LATYSHEV, *On the recognizable properties of associative algebras.* IN: ON COMP. ASPECTS COMM. ALGEBRAS, 237–254. ACAD. PRESS., LONDON, 1989.

ABSTRACT. The paper considers computer algebra in a non-commutative setting. So far, such investigations have been centered on the use of algorithms for equality and of universal properties of algebras. Here, the foundation of all computations is the presentation of the algebra under investigation by a finite number of generators subject to a finite number of defining relations, which satisfy the additional property of forming a Gröbner (or standard) basis. Such algebras are called s.f.p.for standard finite presentation. It is shown that various algebraic properties, such as being finite-dimensional, nilpotent, nil, algebraic, are algorithmically recognisable. When the defining relations are words in the generators, t his is also shown to be the case, for the properties of being semi-simple, prime, semi-prime, etc.

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