American University in Bulgaria

REVIEW

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Regarding: The application of the following candidates for the academic position of **FULL PROFESSOR OF MATHEMATICS** according to the Bulgarian legislation at the American University in Bulgaria in Area of Higher Education 4. Natural Sciences, Mathematics, and Informatics in Professional Field 4.5 Mathematics. The procedure has been announced in the State Gazette issue 69 dated September 8, 2015

Candidate 1: **Assoc. Prof. Dr. Tatyana Gateva-Ivanova**

I present the evaluation below in my capacity as a member of the Academic Jury appointed for the above mentioned procedure by a letter of the AUBG President dated November 6, 2015. This evaluation is based on the *Development of Academic Staff in the Republic of Bulgaria Act*, the Rules for its implementation, the internal AUBG rules, regulations, and policies, including the *Habilitation Procedure* and the advertisement on the AUBG website regarding the procedure.

Evaluation of the Applicants

The evaluation is carried out according to the ***Primary Indicators*** *and* ***Additional Indicators*** *included in the document* ***Guidelines for the Academic Jury.***

I. Basis for Evaluation and general description of the publications and professional activities, submitted for consideration for this procedure.

My evaluation is based on the submitted by the applicant documents and on my personal information because I know the applicant since 1970 and all these years we were Master students and Ph.D. students in the same department, and then we have been working in the same section of the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

The documentation presented by the applicant contains:

- 17 articles in refereed journals and book series specialized in the area of mathematics and mathematical physics (J. Algebra – 6, J. Symbolic Comp. – 2, Algebras and Representation Theory – 2, Adv. in Math., Trans. AMS, Commun. Math. Phys., J. Math. Phys., Serdica Math. J.,

Lect. Notes Comp. Sci. (Springer-Verlag), Progress in Math. (Birkhäuser)); 13 of these articles are with total impact factor 10.555;

- 1 invited article in proceedings of a conference;
- 2 non-refereed publications (preprints posted in arXiv).
- 16 of the publications have not been used for other promotions.
- 11 of the presented publications are without coauthors, 8 are with one coauthor, and 1 is with two coauthors. All coauthors are well known mathematicians – van den Bergh, Majid, Cameron, Latyshev, Jespers and Okniński, Floystad.
- a list of 302 citations of 22 of the publications of the applicant. With a couple of exceptions the citations are by foreign mathematicians, mostly in high level journals (>15 of them are with impact factor). Especially impressive is the number of references of No. 14 from the submitted for the application papers (with 59 citations), and No. 9 (with 48 citations);
- a representable list of visits in respected scientific centers with seminar and colloquium talks, and participations in international scientific forums, also with invited talks;
- a list of personal grants and fellowships including several very impressive: Fulbright Research Scholar at the Massachusetts Institute of Technology; Harvard University; Max-Planck-Institut für Mathematik, Bonn, Germany; L'Institut des Hautes Études Scientifiques (IHÉS), Paris, France; Isaac Newton Institute for Mathematical Sciences, Cambridge, UK; the Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy;
- a list of participations as a member of the scientific team of national and bilateral grants.
- Presented information for advising of Ph.D. and M.Sc. These.

II. Eligibility

The applicant satisfies all the requirements except that she does not have a published monograph. But I think that the scientific contributions of several of the research articles in high level journals are a sufficient compensation.

III. Areas of Research of the Candidate. Evaluation of the Contributions of the Candidate.

The principal area of the research of the applicant is noncommutative combinatorial and computational algebra and its meeting points with other mathematical areas.

The first topic of the research of the applicant is related with the finite presentation of algebras. In commutative algebra, every finitely generated algebra is finitely presented, i.e., it is a homomorphic image of a finitely generated polynomial algebra modulo a finitely generated ideal. Additionally, this ideal has a finite Gröbner basis which allows to recognize algorithmically many important properties of the algebra.

In noncommutative algebra this is not true anymore. In a series of papers the applicant studies algebras which are *standard finitely presented*, i.e., they are finitely presented and the corresponding ideal has a finite Gröbner basis. The main result of **paper No. 20** is that various algebraic properties (being finite dimensional, nilpotent, nil, algebraic) are algorithmically recognizable. The list of such properties is even longer for finitely presented monomial algebras. **Paper No. 19** studies the problem whether the global dimension is effectively computable, gives a positive answer for monomial algebras and reduces the general case to the monomial case. **Papers Nos 18 and 17** consider a class of standard finitely presented algebras with additional structural properties (called *strictly ordered*) and answer completely the problem when such algebras are Noetherian. The answer is very nice. This happens if and only if the algebra is almost commutative and, equivalently, of polynomial growth. Papers Nos 17-20 were used in the habilitation of the applicant. The topic of standard finite presentation continues in **paper No. 15** where the applicant studies a class of quadratic algebras (called *binomial skew polynomial rings*). A series of algebraic properties are established and, as a corollary, a new class of finitely generated noncommutative left and right Noetherian semigroups is constructed.

The main topic of the research of the applicant is the theory of set theoretic solutions of the Yang-Baxter equation. The Yang-Baxter equation (or star-triangle relation) was first introduced in statistical mechanics independently by Yang in 1968 and Baxter in 1971. This is a consistency equation depending on the idea that in some scattering situations, particles may preserve their momentum while changing their quantum internal states. It states that a linear operator r, acting on the tensor square $V\otimes V$ of a vector space V satisfies the condition $r^{12}r^{23}r^{12}= r^{23}r^{12}r^{23}$, where r^{ij} means that r acts on the (i,j)-components of the tensor product $V\otimes V\otimes V$. In one dimensional quantum systems, the matrix of r is the scattering matrix and if r satisfies the Yang-Baxter equation then the system is integrable. The Yang-Baxter equation also shows up when discussing knot theory and the braid groups where it corresponds to swapping two strands. Since one can swap three strands in two different ways, the Yang-Baxter equation enforces that both paths are the same. The Yang-Baxter equation turned out to be the key to revolutions both in knot theory (in the construction of knot invariants) and in mathematical physics (where it was central to a new 'quantum' notion of symmetry, namely a quantum group). It seems that the description of the solutions of the Yang-Baxter equation in the general case is a hopeless problem. In 1991 Drinfeld suggested to study set theoretic solutions of the equation. In the language of linear algebra this means that the operator permutes the tensor products of the elements of the basis of the underlying vector space. The experience of the applicant in the theory of finitely presented algebras allowed her to change smoothly her scientific interests to the theory of the Yang-Baxter equation. *In a series of papers she has systematically developed the purely mathematical theory of set theoretic solutions of the Yang-Baxter equation, has found applications to noncommutative geometry – the Artin-Schelter regularity and to combinatorics. In*

particular, she has discovered a lot of relations with other important algebraic objects and has formulated several conjectures which have motivated active research in the field by other mathematicians. In the moment the applicant is considered to be one of the leading experts in the field. **Paper No. 14** deals with semigroups of *I*-type which appear naturally in the study of Sklyanin algebras. The main results show that *I*-semigroups are related to various other mathematical objects, e.g., to the skew polynomial rings introduced in paper No. 15, in the theory of Bieberbach groups and in the study of set theoretic solutions of the Yang-Baxter equation. **Paper No. 13** is a survey of the recent results (with respect to 2000). In particular, it discusses a conjecture of the applicant on the close relation between nondegenerate involutive set theoretic solutions of the Yang-Baxter equation and a class of standard finitely presented semigroups (called *binomial skew polynomial semigroups*). **Paper No. 12** deals with algebras presented by square-free quadratic relations of the form $x_ix_j = x_kx_l$ with all square-free monomials of second degree participating exactly once. It studies natural problems – when the algebra is Noetherian or satisfies a polynomial identity. The study of the structure of the underlying semigroup gives also information about the prime radical and the minimal prime ideals. In **paper No. 11** the applicant introduces a class of quadratic algebras (called *quantum binomial algebras*) and determines conditions equivalent to the fact that the algebra defines a set theoretic solution of the Yang-Baxter equation. It turns out that such an algebra is of Poincaré-Brikhoff-Witt type and is Koszul, Noetherian and an Artin-Schelter regular domain. The study of square-free involutive solutions continues in **paper No. 10**, where with combinatorial methods the applicant confirms her conjecture that square-free solutions, semigroups of *I*-type, and semigroups of skew polynomial type are equivalent Yang-Baxter structures. In **paper No. 9** the work on square-free solutions is extended to the study with methods of graph theory of their automorphism groups, strong twisted union of solutions, and multipermutation solutions. The study continues in **paper No. 8** in terms of the left and right actions on the underlying set on itself. Part of the results of **preprint No. 7** are published in **paper No. 4**. The authors define the notion of *wreath product* of square-free solutions and show that the wreath product of two multipermutation square-free solutions is also a multipermutation solution. They construct new examples of solutions and establish a series of decomposition results. In **paper No. 6** multipermutation solutions of level 2 are obtained by quantization of polynomial algebras in commuting variables. The paper provides also the first steps in noncommutative differential geometry of the associated Yang-Baxter algebras. **Paper No. 5** shows the intimate connection of square-free solutions of the Yang-Baxter equation with other actively investigated objects – Garside monoids, monoids with quadratic relations, monoids of skew polynomial type. Quadratic algebras of Poincaré-Brikhoff-Witt type of finite global dimensions are studied in **paper No. 3**. The main results show that for algebras with quantum binomial relations and the Poincaré-Brikhoff-Witt property the finite global dimension is equivalent to polynomial growth and to Artin-Schelter regularity. In this way the classification of Artin-

Schelter regular algebras of Poincaré-Brikhoff-Witt type is equivalent to the classification of square-free solutions of the Yang-Baxter equation. In **paper No. 1** the applicant involves new techniques from group theory and the theory of braces in the study of set theoretical solutions of the Yang-Baxter equation. She establishes new criteria for a map to be a multipermutation solution and relates the multipermutation level with the class of nilpotency of the radical ring associated with the corresponding permutation group.

The third topic of the research of the applicant presented for the professorship is on the meeting point of the theory of monomial ideals (in associative algebras) and formal languages, in particular of combinatorics of free Lie algebras. **Paper No. 2** studies monomial algebras defined by Lyndon words. The applicants finds a Poincaré-Brikhoff-Witt basis of the algebra and relates the polynomial growth with the finite global dimension. Especially detailed is the study of the extremal class of Fibonacci-Lyndon algebras. In particular, it is shown that keeping the multigrading one cannot deform one of these algebras to an Artin-Schelter regular algebra.

As I already mentioned, the applicant participates very actively in conferences (including such of high level) and delivers (very often invited) talks. She has a very representative list of grants and fellowships, and talks at world recognized research centers. She was a member of the scientific team of several national and bilateral grants where I was the principal investigator and I have a very high opinion for the activity of the applicant.

Summarizing, I think that Assoc. Prof. Dr. Tatyana Gateva-Ivanova is an active researcher in a modern and important branch of mathematics, where she has obtained excellent results published in high level journals and already recognized by the mathematical community both in Bulgaria and abroad. She is one of the most productive and most cited Bulgarian algebraists.

IV. Evaluation of the Candidate's teaching

My latest direct observations on the teaching abilities of the applicant are from the 1980s when we taught problem solving seminars in linear and higher algebra at the Faculty of Mathematics and Mechanics (now Faculty of Mathematics and Informatics) of the University of Sofia. The classes of Tatyana Gateva-Ivanova were well organized. She presented the material with care for the students, in the same time trying to motivate the students as much as possible. But I have visited many of her research seminar talks, also in English and oriented to general, non-algebraic audience. Her exposition is always very clear, taking into account the level of the audience and, in the same time, keeping the high standard of the exposition. Delivering a talk, the applicant is always very enthusiastic. I can judge from the information presented by the AUBG as a part of the documentation that the teaching abilities of the applicant are very high and her teaching is evaluated as excellent. This is true especially for upper-level courses. For lower-level courses

some of the evaluations are below the Department and University averages, but with tendency for improving. *My believe is that Tatyana Gateva-Ivanova is an excellent teacher both for liberal art universities and for more scientifically oriented institutions. Her teaching abilities completely correspond to the current procedure and will be very useful for the AUBG.*

V. Critical notes and recommendations

I do not have critical notes concerning the research, teaching, and service of the applicant. I have some suggestions and recommendations:

- I think that she has sufficient knowledge and experience to write a monograph on set theoretic solutions of the Yang-Baxter equation;
- I would recommend to be more active in the attracting young people to the field, not only abroad but also at the AUBG and at other Bulgarian universities and scientific institutions. I think that the applicant has the abilities to be a leader of a team, both for the research in the field and for grant applications.

VI. Conclusion:

- *I think that the applicant meets all specific requirements for a professorship at the American University in Bulgaria as well as the requirements of the Development of Academic Staff in the Republic of Bulgaria Act.*
- *I strongly recommend for granting the academic position of Full Professor of Mathematics according to the Bulgarian legislation at the American University in Bulgaria in Area of Higher Education 4. Natural Sciences, Mathematics, and Informatics in Professional Field 4.5 Mathematics to ASSOC. PROF. DR. TATYANA GATEVA-IVANOVA.*

Signature and date:

January 23, 2016

By signing here I also declare that writing this review does not represent conflict of interest.