AUBG MultiTalent Quest
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First Name: _____
ID Number:_____

CharisMATHic Analysis

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Problems

1. What is the probability of throwing one dice three times in a row and get the even number after the first fall, the number greater than four after the second fall and the odd number after the last fall?

A) 3/8 **B)** 3/12 **C)** 1/12 **D)** 1/8 **E)** 1/18

2. A certain price was increased by 40% and then the new price was decreased by 25%. The same final price would be attained by a single increase of the initial price by:

A) 5% **B)** 10% **C)** 15% **D)** 20% **E)** 25%

3. If $2^8+2^8+2^8+2^8=2^n$, then n= **A)** 9 **B)** 10 **C)** 16 **D)** 24 **E)** 32

4. If $x+y+z=2019$, $\frac{x}{y}=\frac{1}{2}$ and $\frac{y}{z}=\frac{111}{170}$, find the value of $z-x$.

A) 678 **B)** 687 **C)** 768 **D)** 786 **E)** 867

5. The parallelogram $ABCD$ has perimeter 144. The angle bisectors of the angles at A and B meet at a point L on the side CD. Find AB.

A)24 B)36 C)45 D)54 E)Another answer

6. What is the median of the following values: 3, -10, 0, 3, 3, 4, -10, 1, 2

A) -4/9 **B)** 3 **C)** 4/9 **D)** 2 **E)** -10

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M1. (**Do this AFTER completing the exam**) Explain the reference in this meme

7. Four of the angles in a pentagon are equal, while the fifth one is twice as small. Find the measure of the largest angle in this pentagon.

A) 60º **B)** 80º **C)** 100º **D)** 120º **E)** 140º

8. How many integers n satisfy the condition $2019 \leq |n| \leq 9102$?

A) 7083 **B)** 7084 **C)** 14166 **D)** 14168 **E)** Another answer

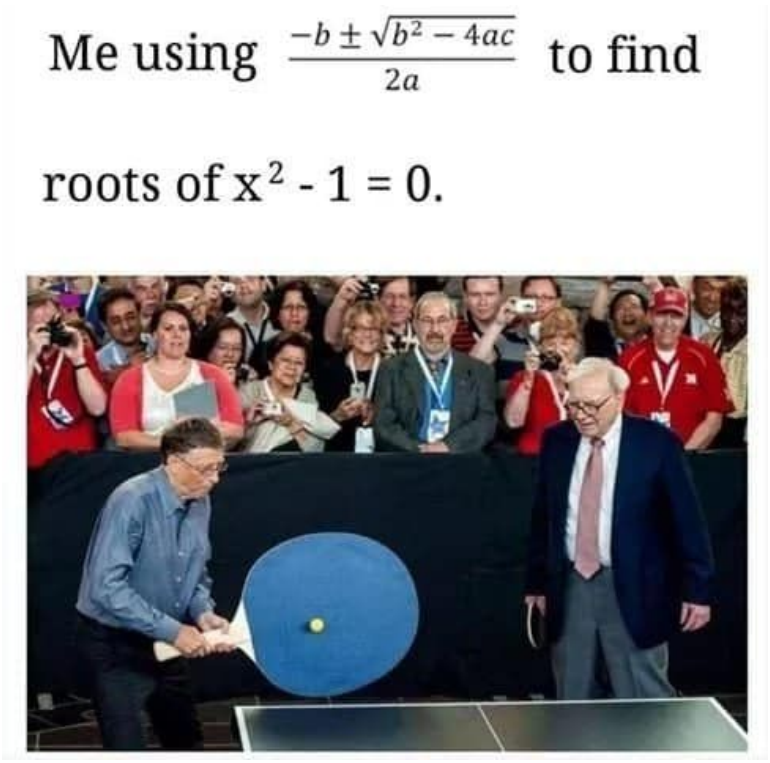
9. Based on the 3Blue1Brow Essence of Calculus video series, which of the following definitions best describes the **intuition (**i.e. conceptual understanding) behind a derivative? Try to motivate your choice.

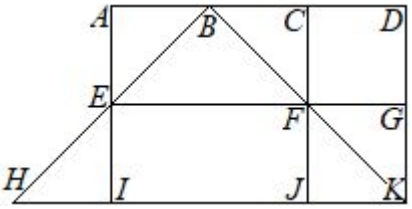
A) The area under a curve **B)** The difference between two values
C) The "acceleration" of a function **D)** The best constant approximation for rate of change
E) A tangent line to some curve

10. A teacher has to choose three students from the class 3A and two students from the class 3B to the recitation competition. There are 22 students in the 3A and 17 students in the 3B class. How many possible choices does she have? [C(X, Y) means combinations of X for Y0]

A) C(22,3)*C(17,2) **B)** C(22,3)+C(17,2) **C)** C(39,5) **D)** C(23,3)+C(18,2) **E)** C(23,3)*C(18,2)

M2. (**Do this AFTER completing the exam**) Do you feel like this now?





11. The picture shows a map of a village (the lines are the streets). You have to put policemen at three of the intersections (denoted with letters) so that each street is under police surveillance (i.e. has a policeman at some of its intersections). There must be a policeman at which of the following intersections:

A) *A* **B)** *B* **C)** *C* **D)** *D* **E)** It cannot be uniquely determined

12. Maria is papering a 5×5 wall using seven square wallpapers of identical size, marked by different symbols. No wallpaper goes beyond the borders of the wall. Some of the wallpapers overlap, so six of them are only partially visible. In what order were they placed?

A)  **B)** 

C)  **D)** 

E) None of the above

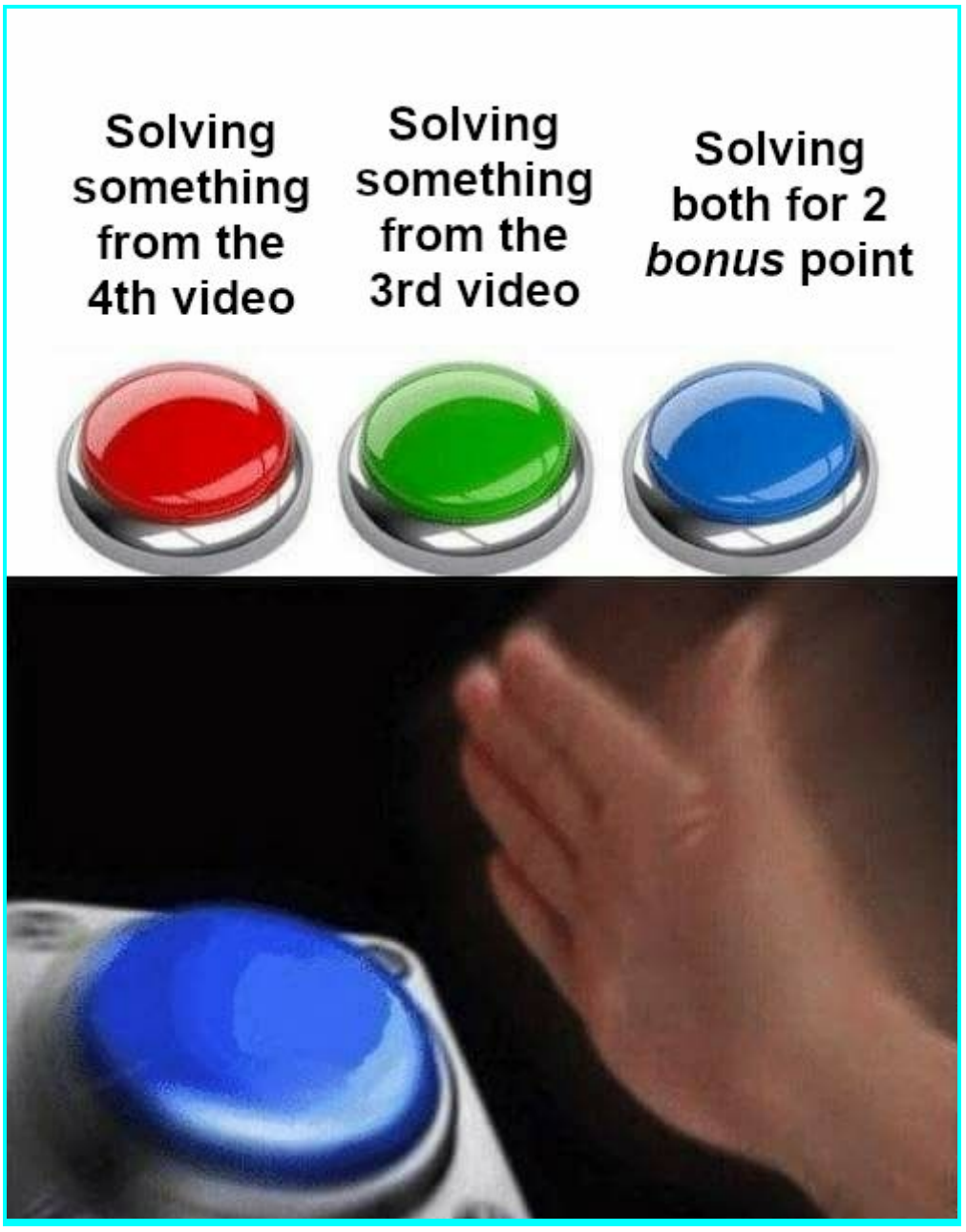
13. The average weight of five boys is 59kg. The average weight of two of these boys is 56kg. What is the average weight (in kg) of the other three boys?

14. One rainy day, a girl broke up with her boyfriend after being together for 5 long years. They decided to part ways where everything about them began at the same time. The boy is due north shaking and crying, running at 3 m/s and the girl is walking due west at a rate of 1 m/s thinking she made the right decision. How fast are they separating from each other after 10 seconds in their new lives without each other?
Hint: This is a related rates problem (i.e. the 6th 3Blue1Brown video).

15. You have to create numbered cards for a sports competition for participants of your school. You are almost done with that but they **change the format** the very last minute! Each numbered card is **3** digits and people from your school can only use the following numbers: *3, 5, 7, 8, 9, 0* (only once, without repetitions). The resulting number has to be divisible by **5** and **cannot** start with **0**. How many such cards can you create?

16. Providing a specific example, that it is possible to raise an irrational number to an irrational power and get a rational solution. Provide the necessary algebraic manipulation to show that this is correct.

M3. Solver **either** problem *17.a* **or** *17.b*. You will be given **full** credit for **only one**, but you will get 2 extra points if you **solve both.**



17.a. What is so special about Euler's number - e? Write down what the main algebraic property of that number is with regard to derivatives $d(e^x)/dx$ **and** write down the formula for the derivative of any power function $d(p^x)/dx$ (using the natural logarithm $ln(a)$).

17.b. Write down the general form of the chain rule of derivatives (i.e. for functions of this kind $f(x) = g(h(x))$) **and** find the exact derivative of this function $f(x) = (sin(x))^2$. For the general form, use $d(f(x))/dx$ to mean the derivative of $f(x)$ with respect to x where x can also be a function like h (the derivative with respect to some function.

18. Find the largest three-digit number that has the same remainder when divided by 5, 6 and 7.

19. Tom is three years younger than Mary and his age is 80% of hers. How many years from now will the age of Tom be equal to 95% of the age of Mary?

20. In the right $\triangle ABC$ the altitude CH to the hypotenuse intersects the angle bisector AL at its midpoint O. If OH=2, find BC.

21. Someone comes up and asks you to prove that any square number (of the form a^2) when multiplied by a number that is 17 more than it ($a^2 + 17$), is always a multiple of 6, else she will tell everyone about that one accident during the winter break. You do not want to risk it.

M4. (**Do this AFTER completing the exam**) Caption this.

(Note, the low resolution is intentional)

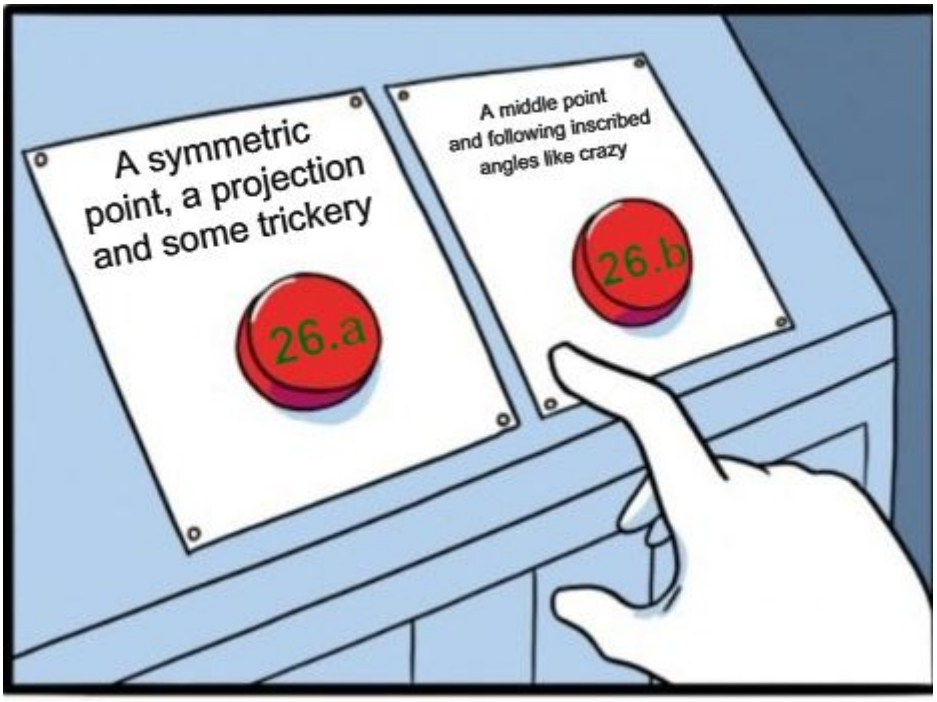
22. You risked and failed your English literature exam (Who would have guessed all the answers were false!?!) but you have a makeup tomorrow. It has 8 questions, each of them being true-false. You still feel lazy. You need 40 percent correct answers or better. Calculate the probability of getting a passing score by guessing, again. Are you going to risk it this time?

23. Your friend is about to have a calculus quiz on derivatives and he is completely confused at the concept of derivatives. You decide to channel your inner 3Blue1Brown and help your friend. Give a visual proof, like those found in the videos, of what the derivative of $\sqrt{x}$ is.

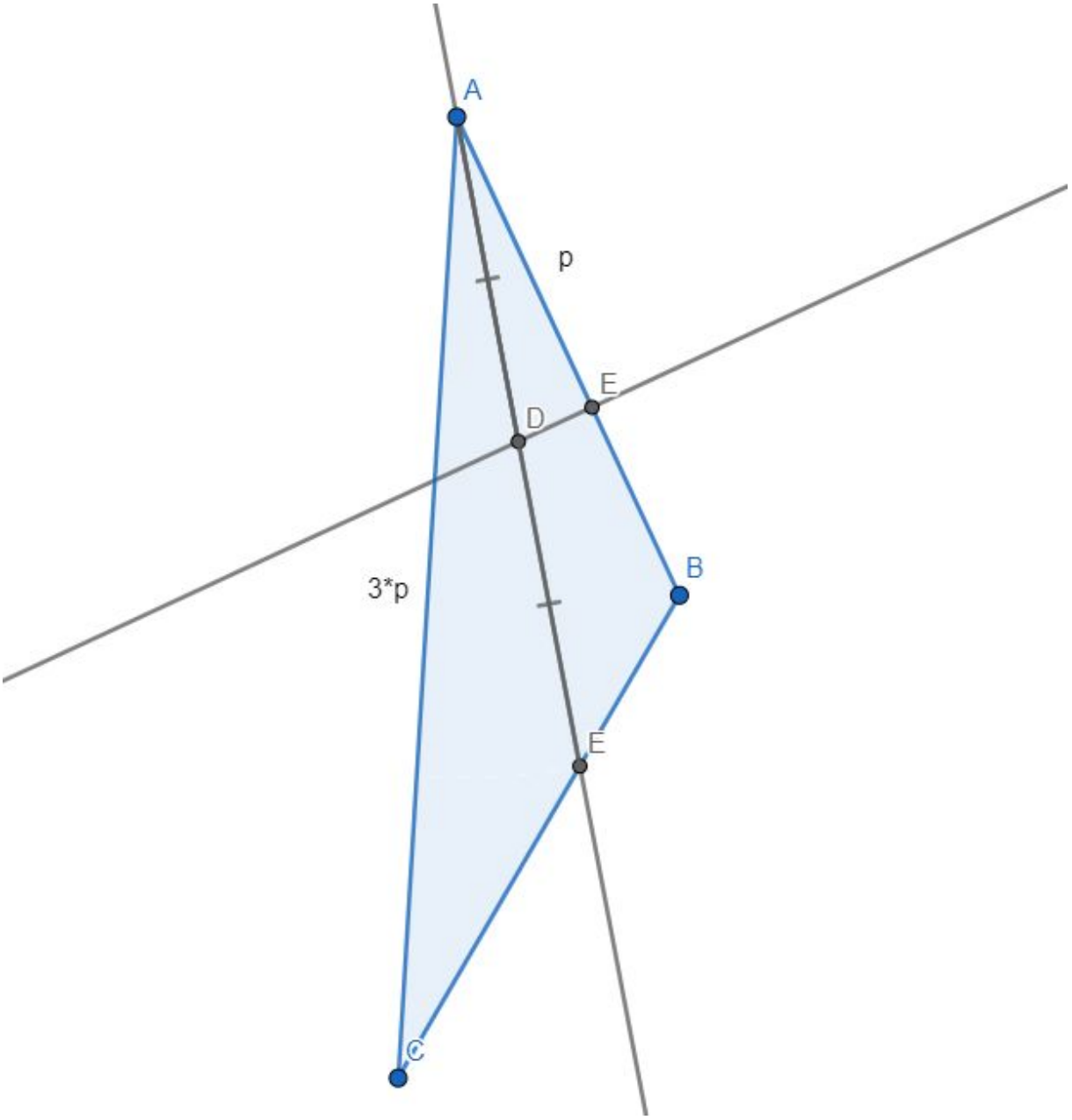
24. Let y be a parabola with two distinct solutions and a leading coefficient 1. Draw a square, using the entire line segment cut by the parabola from the x-axis as one of its sides. Prove that the area of the square equals the discriminant of y.

25. In a data set of 5 elements, the mean, mode and median are all $\sqrt{18}$, the range is $\sqrt{50}$ and the IQR (interquartile range) is $\sqrt{2}$. Find all 5 elements.

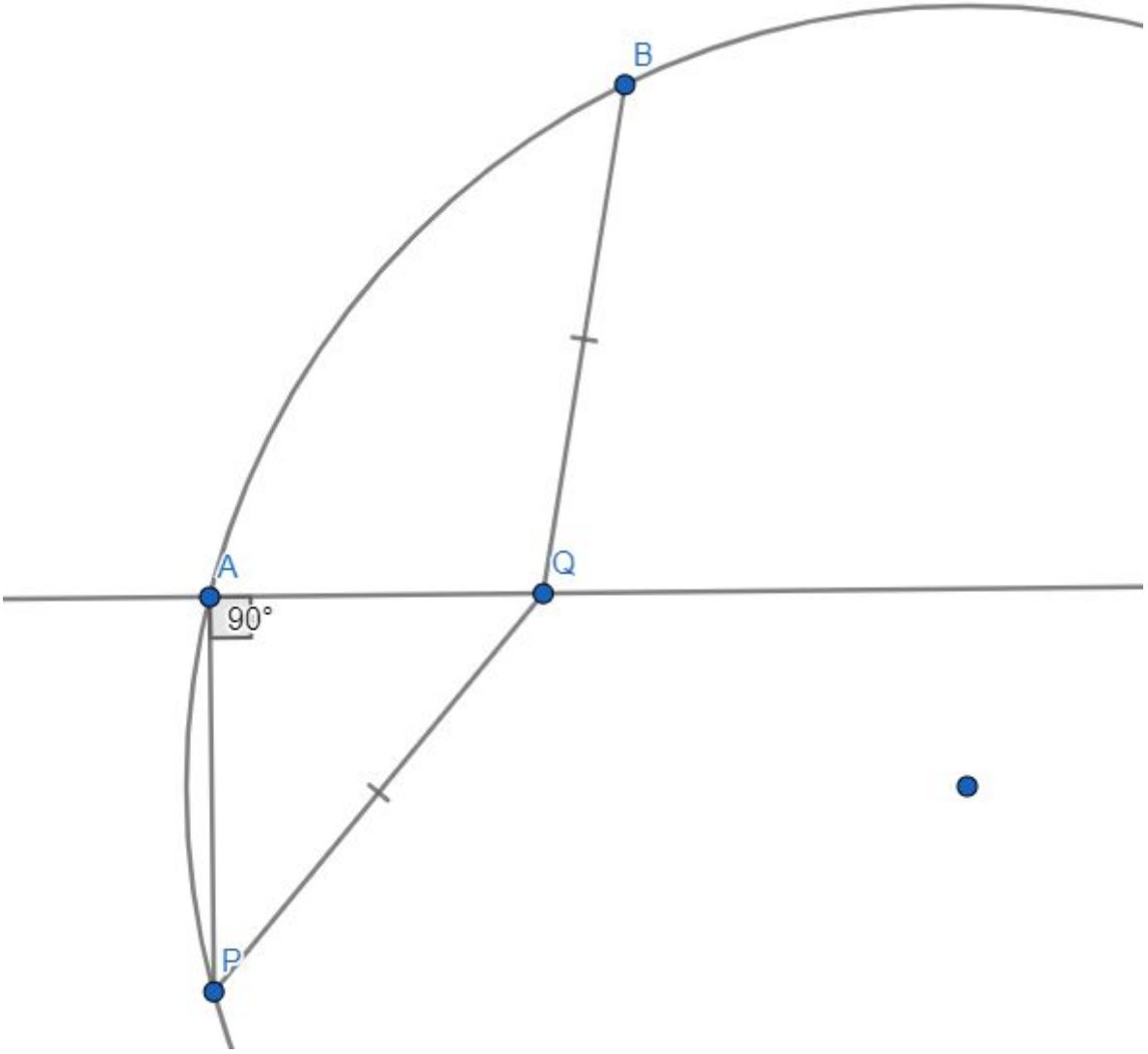
M5. Solver **either** problem *26.a* **or** *26.b*. You will be given credit for **only one**.



26.a. Let AL be the bisector of ΔABC, point D — its middle, E — projection of D on AB. Given that AC=3AE. Prove, that ΔCEL is isosceles.



26.b. Points P, A, B are located on the circle, inside the circle is point Q, such that ∠PAQ=90, PQ=QB, and points P and B are lying on opposite sides of AQ. Prove that ∠AQB-∠PQA=2∠APB.



M6. **(Do this AFTER completing the exam)**

You are at a party organized by the MAT and ECO departments at AUBG and you meet this attractive ECO major you want to impress with your vast mathematic knowledge. The two of you were taking calculus so you go there and say the following:
- (You) [Said with an enquiring and serious voice] All these lectures on derivatives we had to go through really make me want to be one of them. [Pause]
- Oh, why is that? [Generally intrigued]
- So I could lie tangent to your curves.
- [Laughs] Oh, that is sweet, but I have heard that one. You know what, if you can think up of something else that will make me laugh, we can go for dinner some time.

Thankfully, you just watched some videos on Calculus! What more do you say?

AUBG lingo: ECO major – a student majoring in economics; MAT – a student majoring in mathematics.