AUBG MultiTalent Quest
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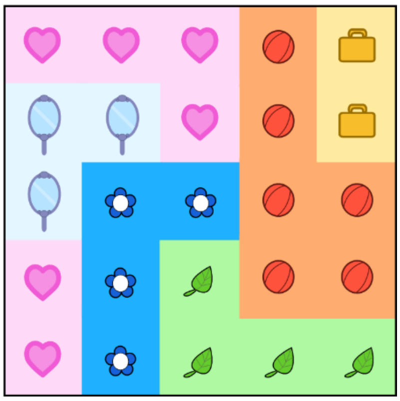
CharisMATHic Analysis
February 17, 2018
Problems

1. If x and y are odd integers, then which of the numbers below must be also odd?

A) xy **B)** $x-y$ **C)** $x+y$ **D)** $x+3y$ **E)** $xy+y$

2. Ann is papering the 5×5 wall using rectangle wallpapers of different sizes. The wallpaper is always placed so as not to go beyond the borders of the wall. She uses a different color (shown with different symbols) for each rectangle of wallpaper. Sometimes, she covers part of the wallpaper with a new rectangle of a different color. In which order did Robyn place the wallpaper?



A)  **B)** 

C)  **D)** 

E) None of the above

3. How many integers n satisfies the condition $3\le|n|\le9$?

A) 6 **B)** 7 **C)** 10 **D)** 12 **E)** 14

4. If $4^8+4^8+4^8+4^8=4^n$, then $n=$ **A)** 9 **B)** 12 **C)** 16 **D)** 24 **E)** 32

5. Points M, N lie on the circumference of a circle c. If $MN=6$, find all possible values of the area A of the circle c.

A) $A=9\pi$ **B)** $A\ge6\pi$ **C)** $A\ge9\pi$ **D)** $A>6\pi$ **E)** $A>9\pi$

6. *If* $3x+5y=a$ and $3x-5y=b$, then $xy=$?

A) $\dfrac{a^2-b^2}{15}$ **B)** $\dfrac{a^2-b^2}{30}$ **C)** $\dfrac{a^2-b^2}{60}$ **D)** $\dfrac{a^2-b^2}{120}$ **E)** $\dfrac{a^2-b^2}{225}$

7. The sum of six consecutive integers is 999. The largest one among these integers is:

A) 166 **B)** 167 **C)** 168 **D)** 169 **E)** 170

8. Three of the angles in a quadrilateral are equal, while the fourth one is three times smaller. Find the measure of the largest angle in this quadrilateral.

A) 54º **B)** 96º **C)** 108º **D)** 120º **E)** 144º

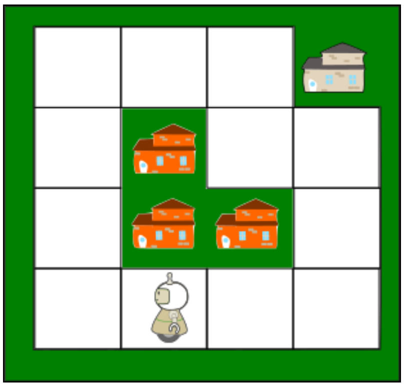
9. In triangle ABC the obtuse angle between the angle bisectors of angles B and C measures 127º. Find the measure of angle BAC.

A) 53º **B)** 54º **C)** 63º **D)** 74º **E)** 83º

10. A straight line passes through the points $A(1;8)$, $B(3;2)$ and $C(6;y)$. Find y.

A) -8 **B)** -7 **C)** -6 **D)** -5 **E)** -4

11. Robot-policeman works in town and every night does its rounds along the street (see the map). Robot-policeman uses only three operations: Forward, Left_Turn (without moving forward), Right_Turn (without moving forward). What is the minimum number of operations needed by robot-policeman to do one round along the street, ending on the same place where it started?



A) 12 **B)** 14 **C)** 16 **D)** 18 **E)** 20

12. Sabina has blue beads and twice as many red beads. Some of the red beads are in a jar, while all other beads are in a box. The ratio of red to blue beads in the box is $2:9$. Find the ratio of the quantities of red beads in the box and in the jar.

A) $1:5$ **B)** $1:6$ **C)** $1:7$ **D)** $1:8$ **E)** $1:9$

13. Find the area of the triangle, whose vertices have coordinates $(0;0)$, $(16;30)$ and $(-30;16)$.

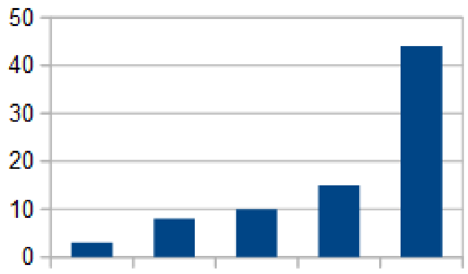
A) 578 **B)** 608 **C)** 768 **D)** 960 **E)** more than 999

14. Let n be the least positive integer such that when n is divided by 5, the remainder is 1; when n is divided by 6, the remainder is 2; and when n is divided by 7, the remainder is 3. What is the remainder when n is divided by 11?

A) 5 **B)** 6 **C)** 7 **D)** 8 **E)** 9

15. Jack was counting the passing cars of different colors and obtained the list: Blue: 10; Red: 44; White: 3; Green: 8; Black: 15. Jack typed the list into a computer program which produced the diagram on the right. However, the columns did not show the colors of the cars. Which color car does the middle column in the table represent?

A) Blue **B)** Red **C)** White **D)** Green **E)** Black



16. If $x+y+z=791$, $\frac{x}{y}=\frac{10}{9}$ and $\frac{y}{z}=\frac{6}{25}$, find y.

A) 108 **B)** 126 **C)** 144 **D)** 162 **E)** 168

17. How many hours are needed to mow a square field of perimeter 1 km, if it is mowed at a rate of 125 sq.m per hour?

A) 64 **B)** 128 **C)** 250 **D)** 256 **E)** 500

18. A certain price was increased by 60% and then the new price was decreased by 15%. The same final price would be attained by a single increase of the initial price by:

A) 33% **B)** 36% **C)** 39% **D)** 42% **E)** 45%

19. Find the perimeter of an isosceles triangle that has one side of length 7 and another of length 17.

A) 41 **B)** 34 **C)** 31 **D)** 24 **E)** It cannot be determined

20. The average of the measures of five of the angles of an octagon is 159º. What is the average measure of the remaining three angles of this octagon?

A) 95º **B)** 105º **C)** 115º **D)** 125º **E)** 135º

21. The average weight of six boys is 59 kg. None of them is heavier than 67 kg, and the weights of each two differ by at least 2 kg. What is the least possible weight (in kg) of one of these boys?

22. The area of a circle of radius 27 is greater than the area of a circle of radius 15 by what percent?

23. To paint the walls and the ceiling of a rectangular room of length 600 cm and width 400 cm one needs 7.3 kg of paint. If one square meter needs 0.1 kg of paint, find the height of the room (in centimeters).

24. Point C lies on a circle with diameter AB and area 289π. Find the area of triangle ABC if $AC = 16$.

25. Ann put $1000 for one year in bank with an annual interest of 6%. Ben put $1000 for one year in bank with an interest of 3% for each six months. What is the difference of their accounts after one year (in cents)?

26. Find the largest three-digit number that has remainder 13 when divided by 16 and remainder 8 when divided by 11.

27. A car traveled with an average speed of 96 km/h from Sofia to Blagoevgrad. If on the way back its time was increased by 20%, then its average speed has decreased by how many km/h?

28. A number sequence consists of the numbers 32, 16, 8, 4, 2, 1 and after that these 6 numbers are repeated forever. Find the sum of the 83^{th}, 84^{th} and 85^{th} number in this sequence.

29. Mary is six years younger than Tom. Tom is now four times older than Mary. How many years from now will the age of Mary be equal to 80% of the age of Tom?

30. Pat has won only 3 of his first 20 games in table tennis. What is the least number of additional games that he needs to play, knowing that he will lose at least 40% of them, in order to have won more than half of his games?

31. For a moment image that you are called Hippasus and live in the ancient Greek colony Metapontum. You also happen to be a Pythagorean philosopher. One day you decide to find the ratio of which two integers p and q you can represent the number $\sqrt{2}$. However, you seem to be stuck and even consider this to be impossible.

Thus, you try to show that **no** such integers exist such that $\frac{p}{q} = \sqrt{2}$.

Note: Try reasoning about division properties of numbers.

Life advice: After completing the proof, better not share it with your fellow Pythagoreans.

32. You are strolling through Turin in the 1890s and a guy named Peano comes and tells you that he has found a solid foundation for mathematics. But he wants to go on to define his Latine sine Flexione ("Latin without Inflection") language so that people can communicate in a unified language. Thus, needs your help to prove something.

Natural numbers are defined as followed. You have a constant called Z (also know as 0) and a function called successor S (think of this as adding +1 to the number) which gives you the next largest natural number. So, S (Z) is the number after zero (1), S(S(Z)) is 2 and so on. We define addition of natural number as follows:
Let m and n be natural numbers. Define m + n as:

- If $m = 0$, then m + n = 0 + n = n
- If $m \neq 0$, thus m = S (k), (k is a natural number), then m + n = S (k) + n = S (k +n)

  [The above is a recursive operation, i.e. we do it until we reach a base case]

Using the above definition show that m + n = n + m.

Note: The above definition is inductive so try an inductive proof. A reminder, an inductive proof means that you must have a base case such that it holds for zero and a way to show that it holds for the next element (the plus one).
Note 2: You can assume that m + 0 = m as well (even though this also needs a proof).

33. Consider yourself a young mathematics student at Göttingen in the 1920s. You meet Emmy Noether and she thinks you might be a good candidate to become one of her student assistants on this new approach to doing algebra, so she decides to test your abstract reasoning.

Imagine you have a set of values (x, y, z, etc.), an operation *op* which takes two elements and produces a third element (*op(x, y) = z*), an operation *inv* which takes an element and produces a different element (*inv(x) = y*) and a constant (an element of the set) called *e*. In this set, the following axioms hold:

- *op(x, op(y, z)) = op(op(x, y), z)* (associativity, i.e. you can reorder braces)
- *op(x, e) = op(e, x) = e* (identity, i.e. *e* does not change anything)
- *op(x, inv(x)) = op(inv(x), x) = e* (cancellation, i.e *inv(x)* is the opposite of x)

Using the above, show that for any elements in this set *a* and *b*, the following holds true

- *inv(op(a,b)) = op(inv(b), inv(a))*

34. February 15, 2018, you tell your grandma that you cannot come and eat because you have a mathematics exam in two days and you want to revise the proofs around the Pythagorean theorem. She mentioned she has forgotten the theorem. Now you have to explain the theorem to your grandma.

Give a visual proof of the Pythagorean theorem $a^2 + b^2 = c^2$. Try to reason about how it relates to a right-handed triangle and the area it expresses.

35. You are in the club and this guy challenges you to prove that $x^n + y^n = z^n$ has no integer solutions for $n > 2$ and $x, y, z \neq 0$. How do you respond?

36. You are at party organized by the MAT and ECO departments at AUBG and you meet this fine ECO major you want to impress with your vast mathematic knowledge. What do you say?

AUBG lingo translation ECO major – a person majoring in economics; MAT – short for anything mathematic related.